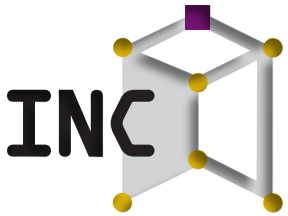
Network Coding: An Overview



Raymond W. Yeung

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Department of Information Engineering

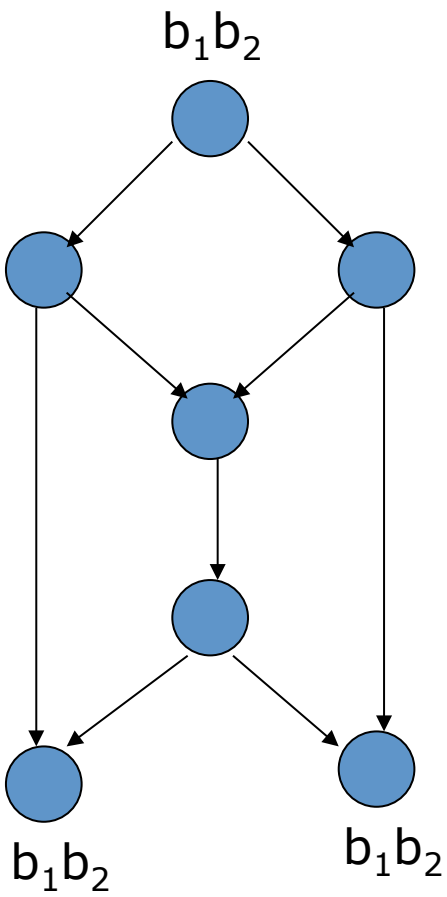
The Chinese University of Hong Kong (CUHK)

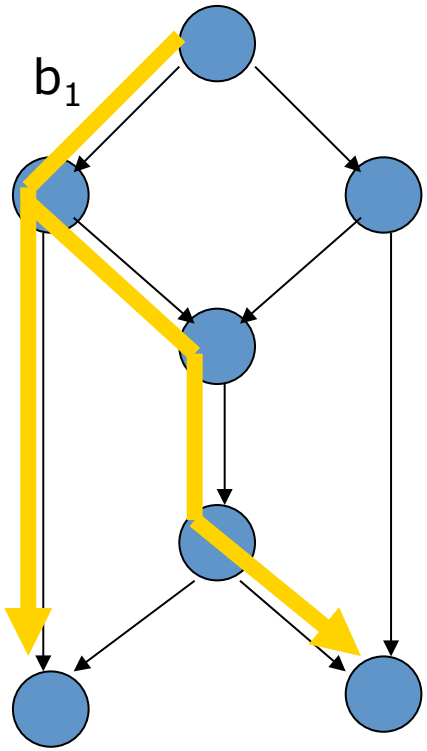
Outline

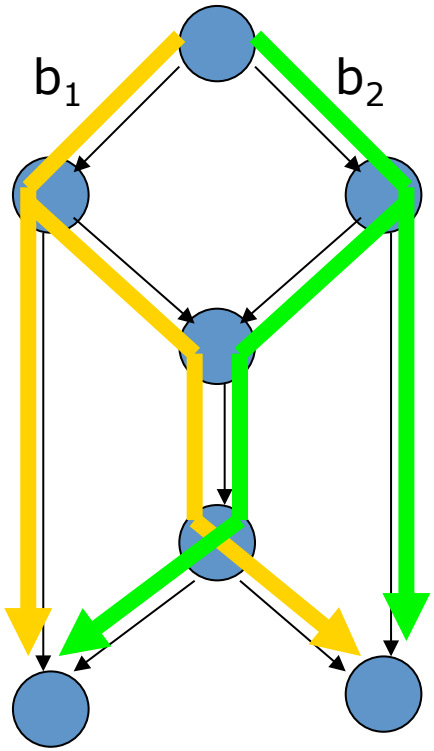
- Introduction and Examples
- Single-Source Network Coding
- Extensions of Network Coding
- Concluding Remarks

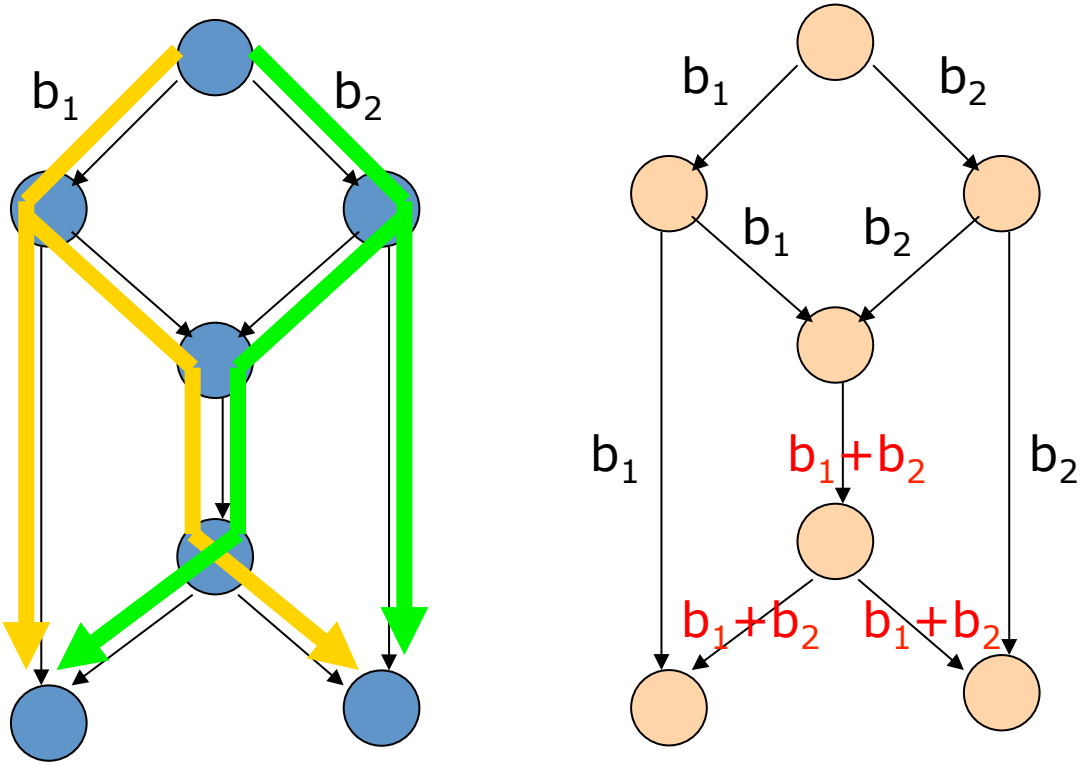
A Network Coding Example

The Butterfly Network


b_1b_2
b_1b_2
b_1b_2

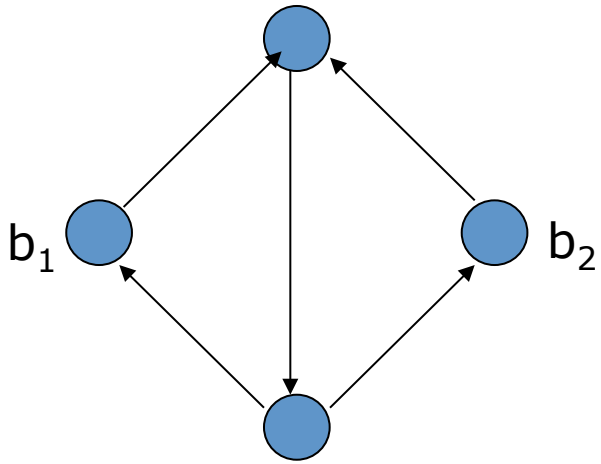

b_1

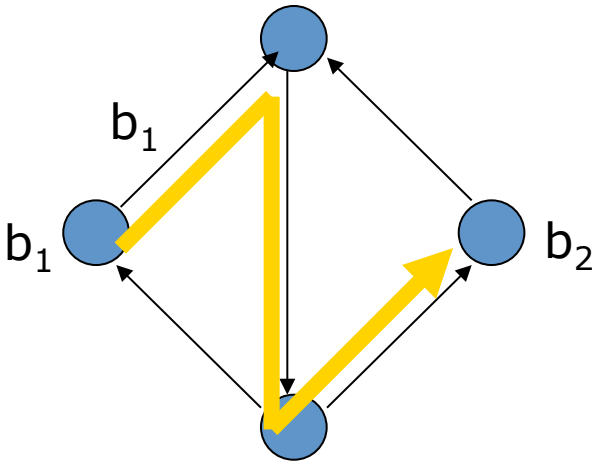

b_1
b_2


b1
b2
b1
b2
b1
b2
b1
b2
b1+b2
b2
b1+b2
b1+b2

A Network Coding Example

with Two Sources


b_1
b_2


b1
b1
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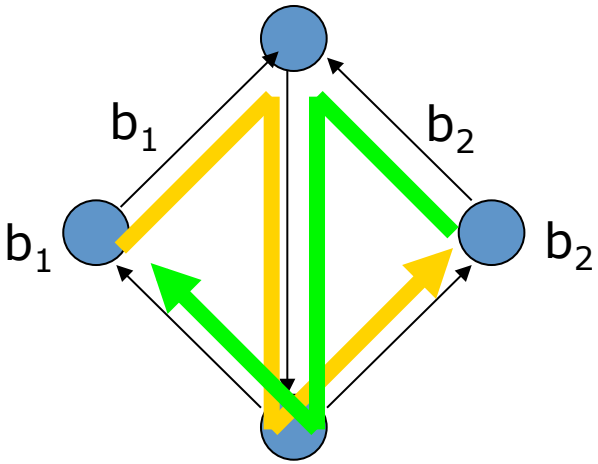
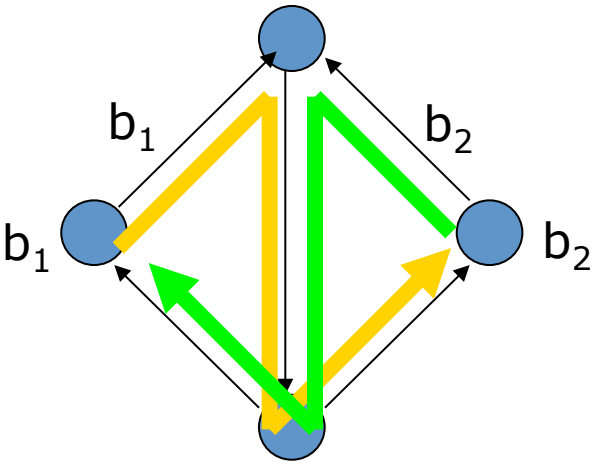
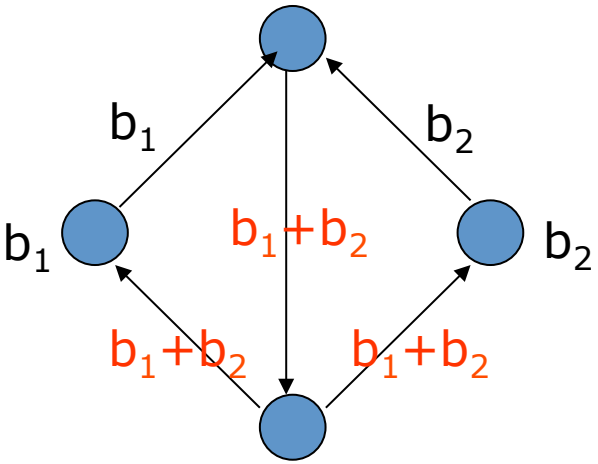
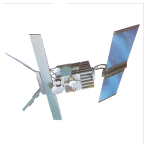
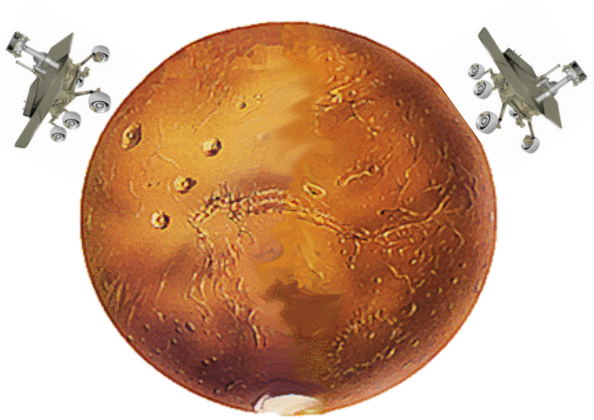

b1
b2
b1
b2


b1
b2
b1
b2


b1
b2
b1
b1+b2
b2
b1+b2
b1+b2

Satellite/Wireless Application





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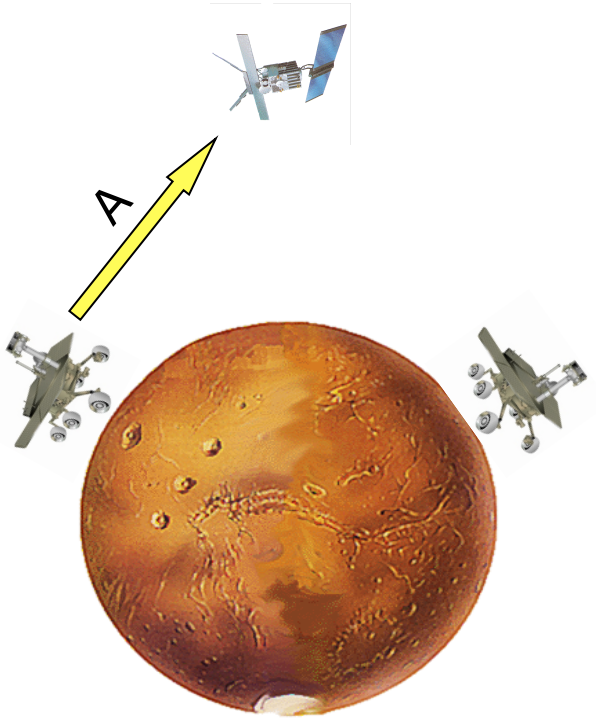

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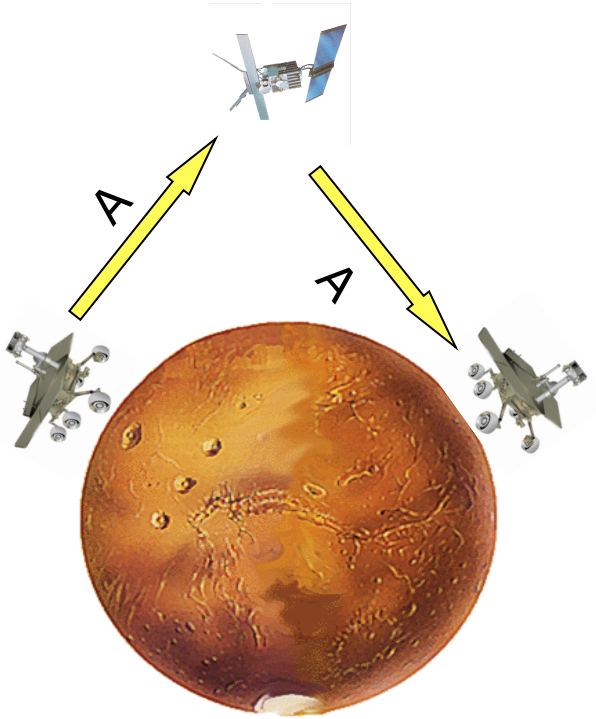

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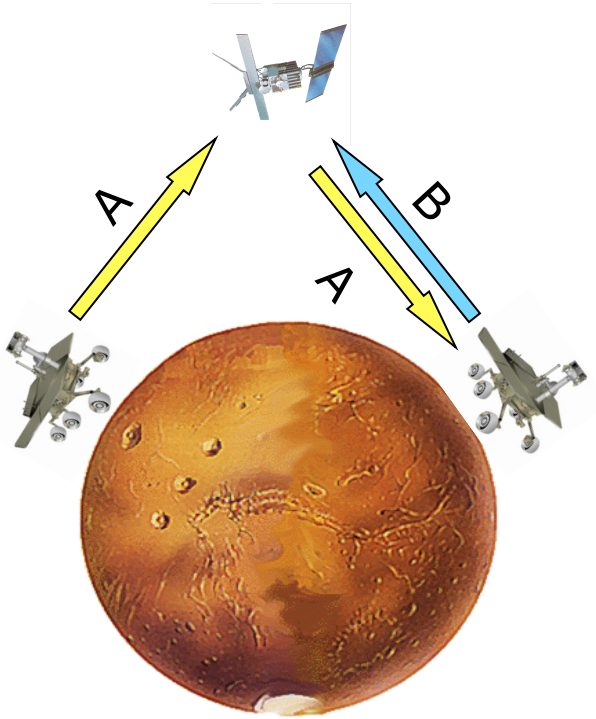

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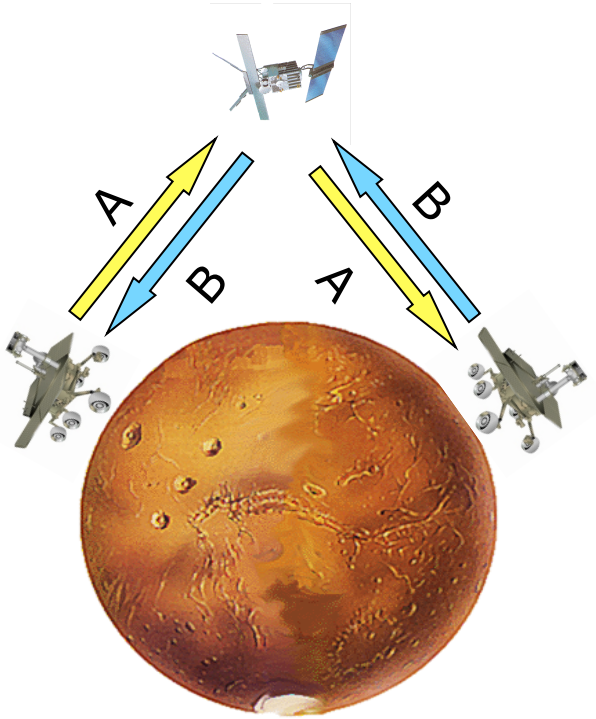

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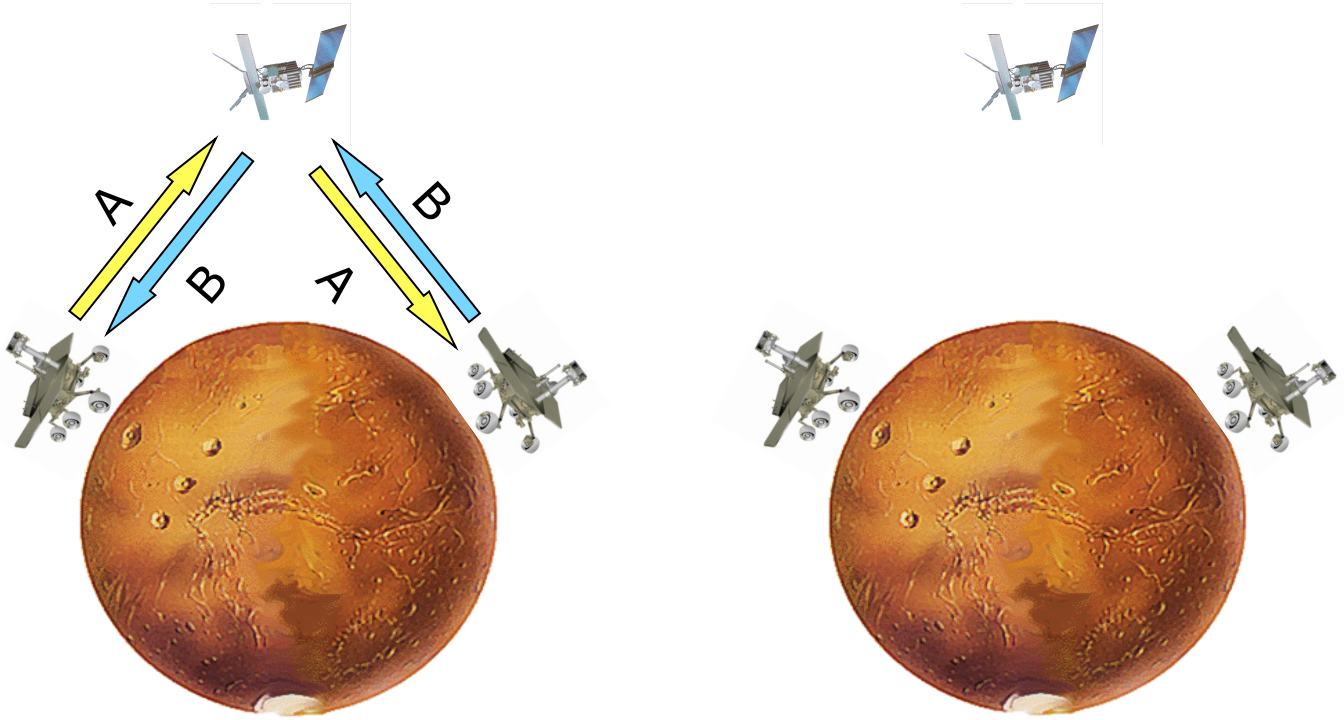

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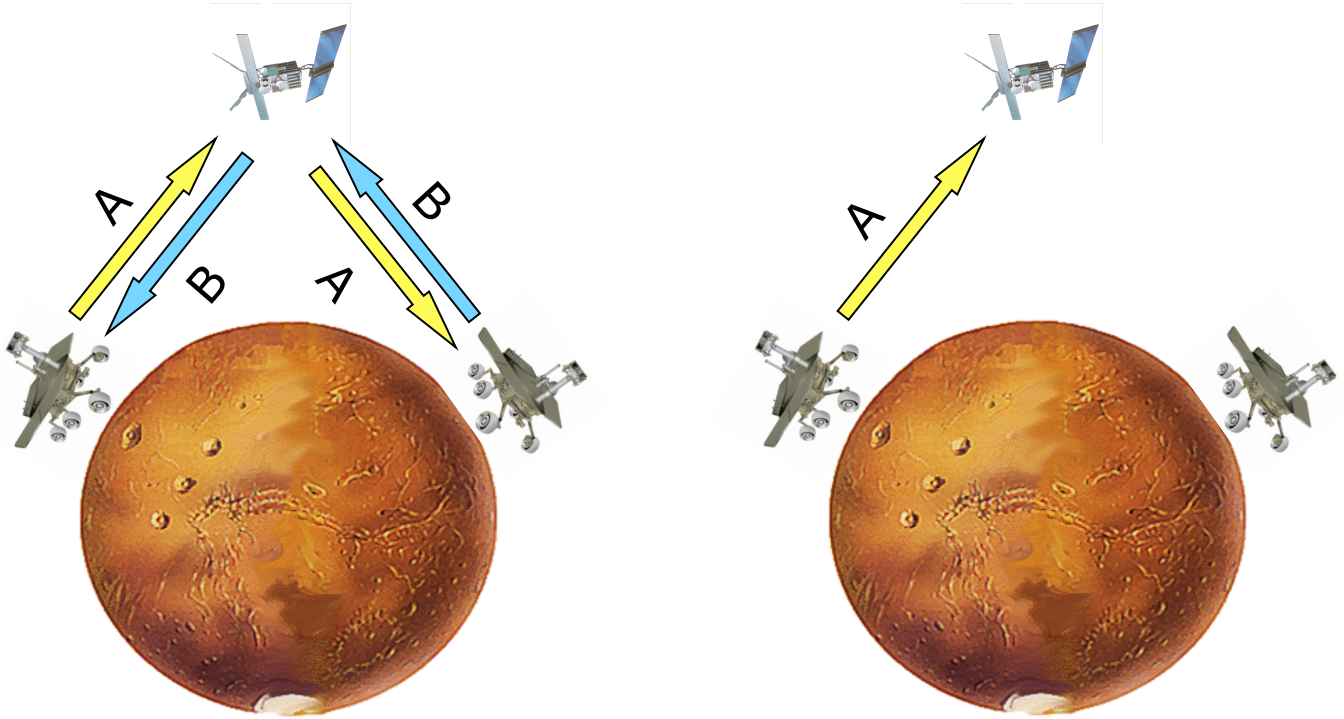

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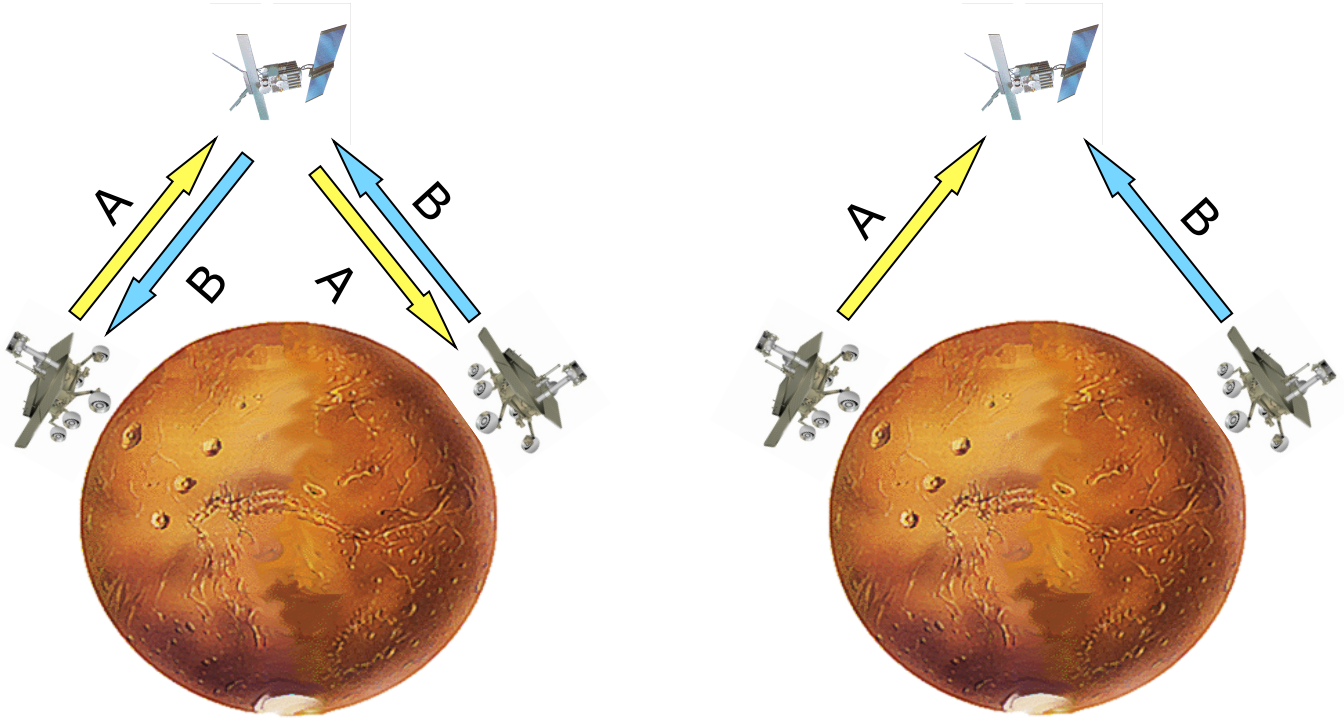

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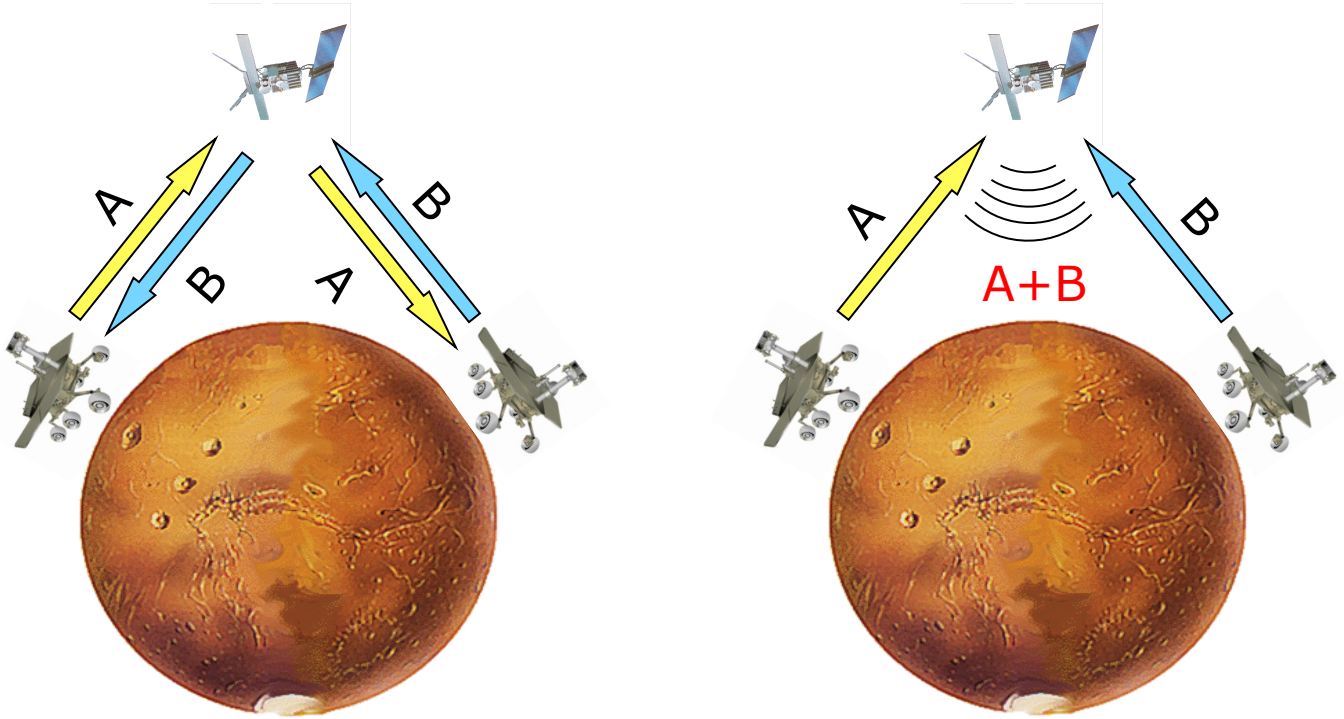


Satellite/Wireless Application

- NASA project proposal (2008)
- Katti et al. (2006/2008) implemented on 802.11 at MAC layer (COPE)

Two Themes of Network Coding

- When there is 1 source to be multicast in a network, store-and-forward may fail to optimize bandwidth
- When there are 2 or more independent sources to be transmitted in a network (even for unicast), store-and-forward may fail to optimize bandwidth

Two Themes of Network Coding

- When there is 1 source to be multicast in a network, store-and-forward may fail to optimize bandwidth
- When there are 2 or more independent sources to be transmitted in a network (even for unicast), store-and-forward may fail to optimize bandwidth
- In short, Information is NOT a commodity!

Single Source vs. Multiple Sources

- Single-source network coding
  - Explicit characterization by Max-flow Min-Cut Theorem for information flow (graph-theoretic)
  - Numerous applications are emerging

Single Source vs. Multiple Sources

- Single-source network coding
  - Explicit characterization by Max-flow Min-Cut Theorem for information flow (graph-theoretic)
  - Numerous applications are emerging
- Multi-source network coding
  - Implicit characterization in terms of achievable entropy functions (Yan, Yeung, Zhang, 2007)
  - Still at the stage of theoretical research

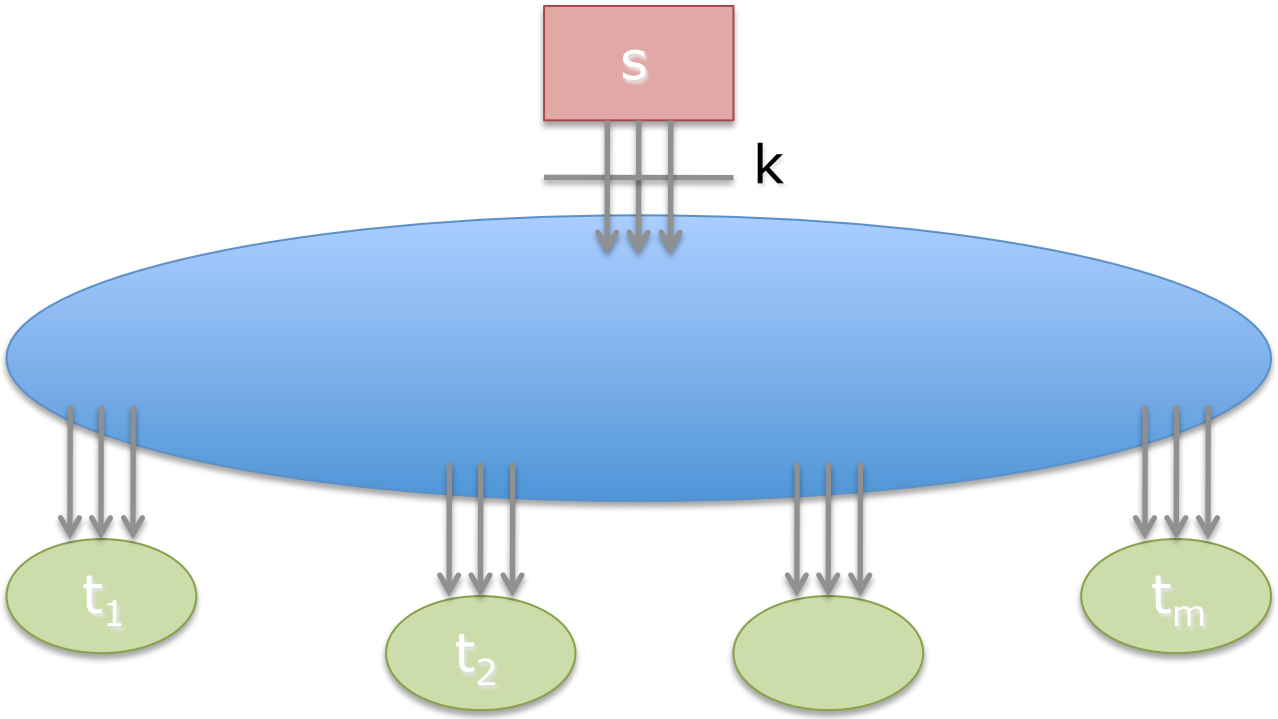
Single-Source Network Coding

Max-Flow Min-Cut: Commodity Flow

- Elias, Feinstein, and Shannon (1956)
- Ford and Fulkerson (1956)

Maximum flow = Minimum cut

Max-Flow Min-Cut: Information Flow



Max-Flow Min-Cut: Information Flow

- Ahlswede, Cai, Li, and Yeung (1998/2000)

Rate = k is achievable
by means of network coding

iff

maxflow(s,t_i) ≥ k
for i = 1, 2, …, m

Linear Network Coding

- Linear network coding suffices
  - Vector space approach: Li, Yeung and Cai (1999/2003)

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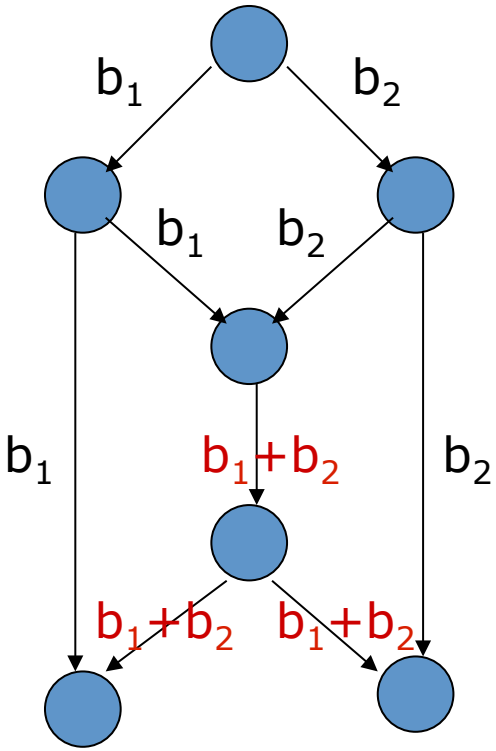
- A sufficiently large finite field chosen as the base field

Example: Butterfly Network



k = 2

F = GF(2)

Random Linear Network Coding

- Ho, Koetter, Medard, Karger, Effros (2003/06)

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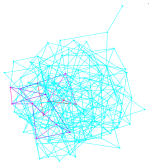
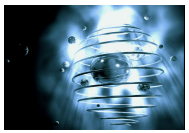
- Can decode w.p.≈ 1 provided that the base field is sufficiently large
- Enables network coding in unknown network topologies



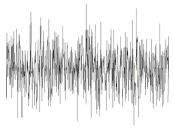
Graph theory



Quantum information theory

Information theory

Channel coding



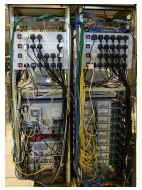
Wireless networks

Optimization theory

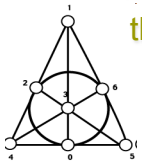
Computer networks

Game theory

Switching theory



Matroid theory

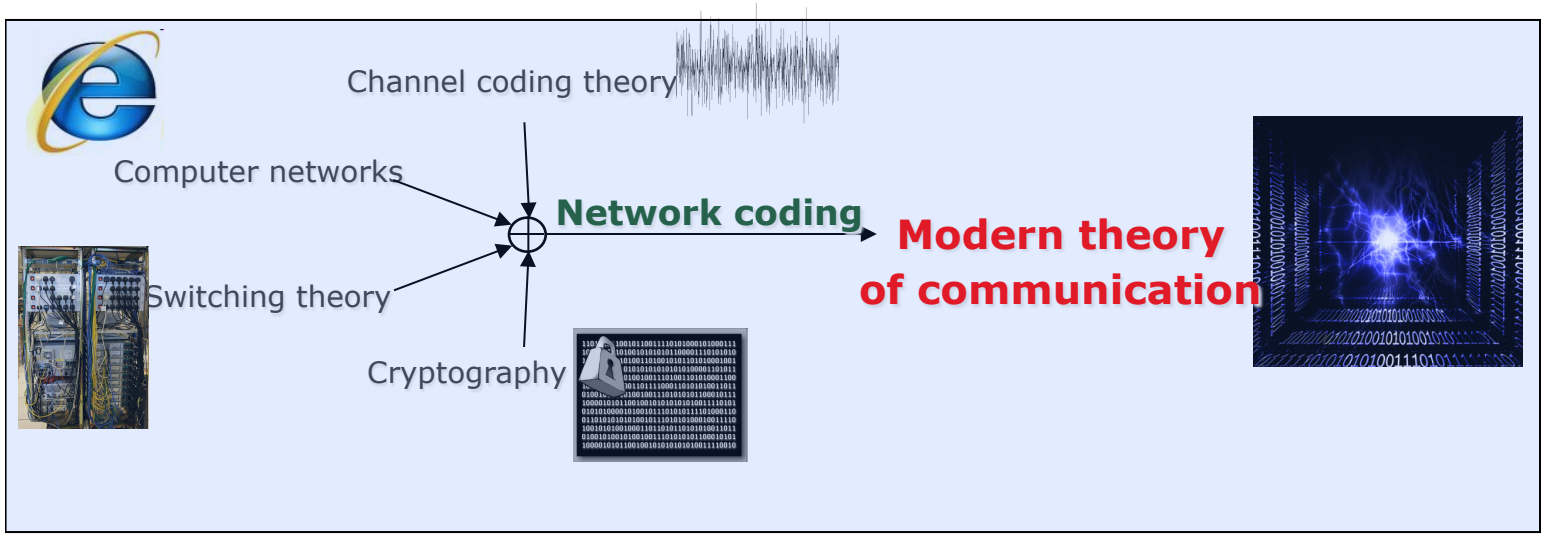


Cryptography

Computer science

Data storage

Network Coding Roadmap



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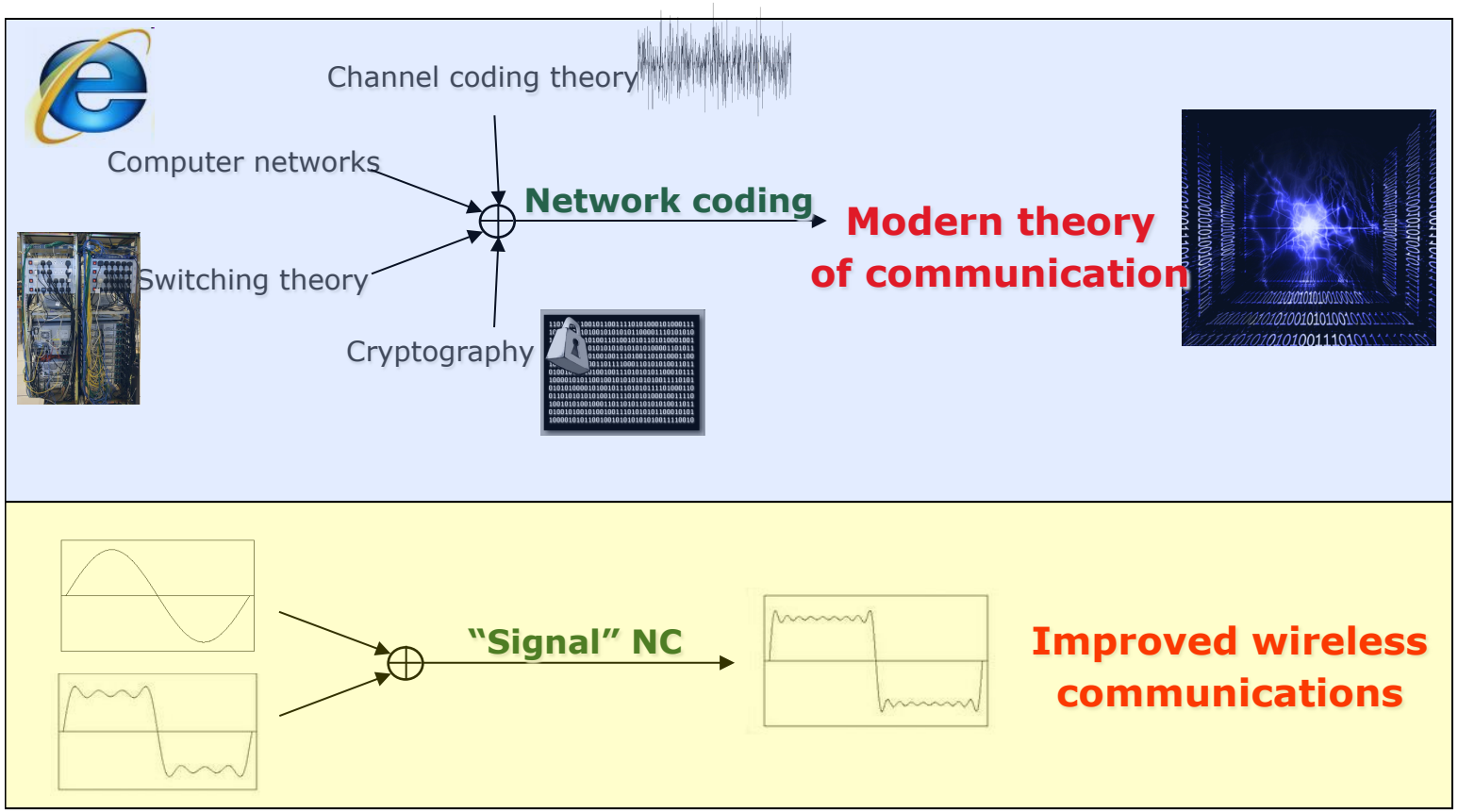

Network Error Correction

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  - Bounds: Hamming, Gilbert-Varshamov, Singleton
  - Network Singleton bound achievable
- Can correct random errors and neutralize malicious nodes

Secure Network Coding

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- Uses network coding against wiretapping

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- Subsumes secret sharing in cryptography
- Information-theoretic bounds achievable for some important special cases

Signal-Level Network Coding

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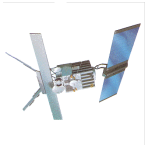
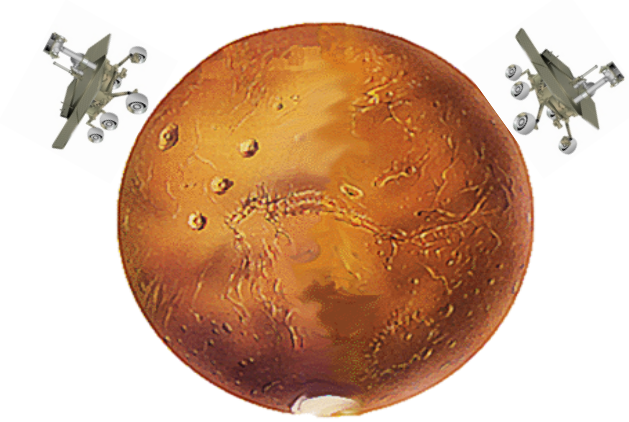
- Physical-Layer NC: Zhang, Liew, and Lam (2006)
- Analog NC: Katti, Gollakota, and Katabi (2007)

Illustration of PNC/ANC





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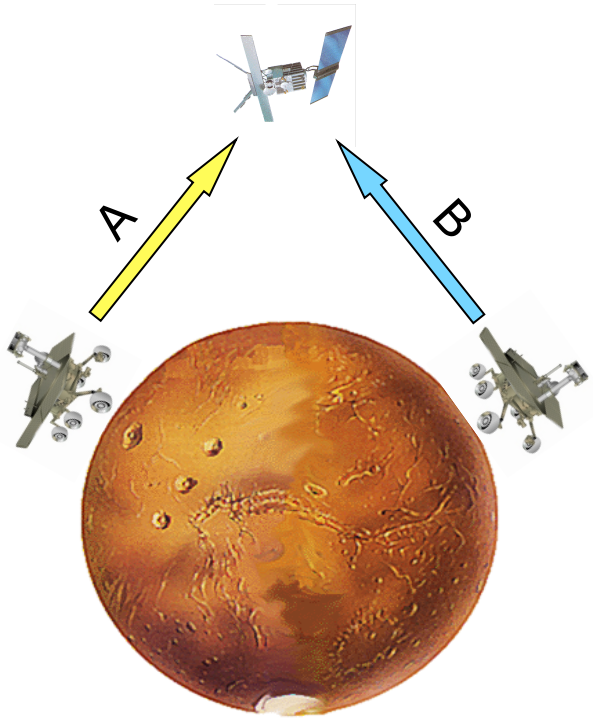

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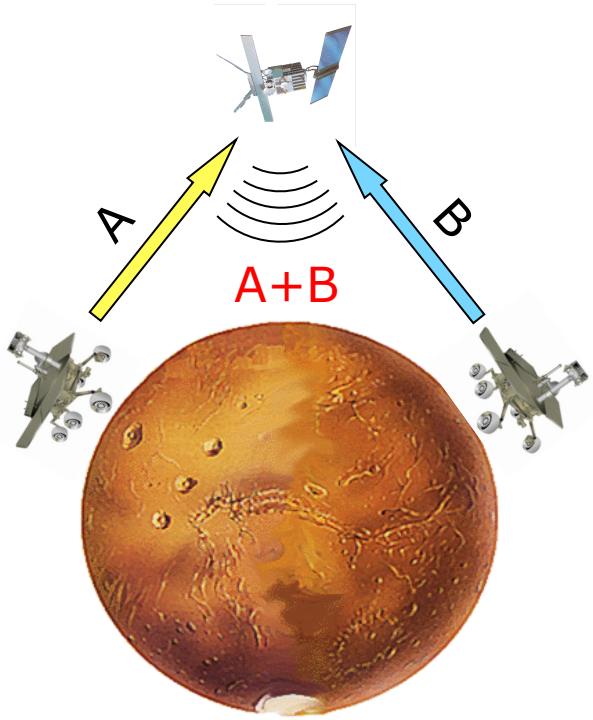

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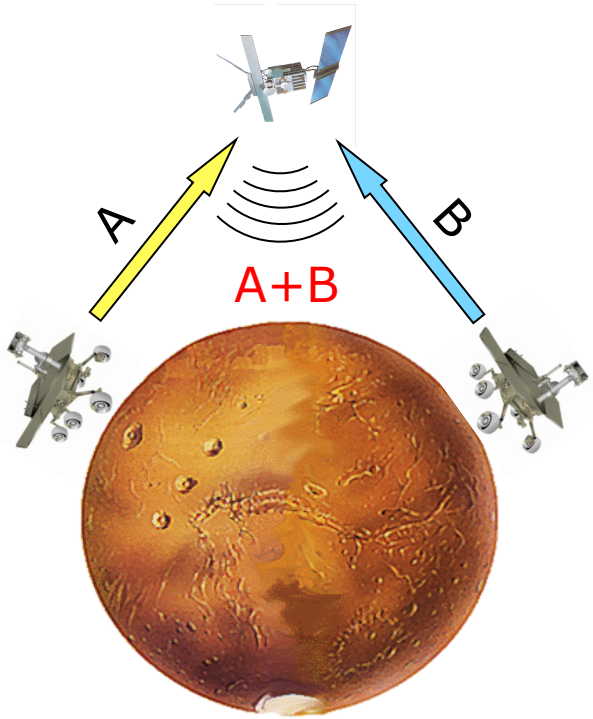

PNC (CUHK)

- Estimates A+B

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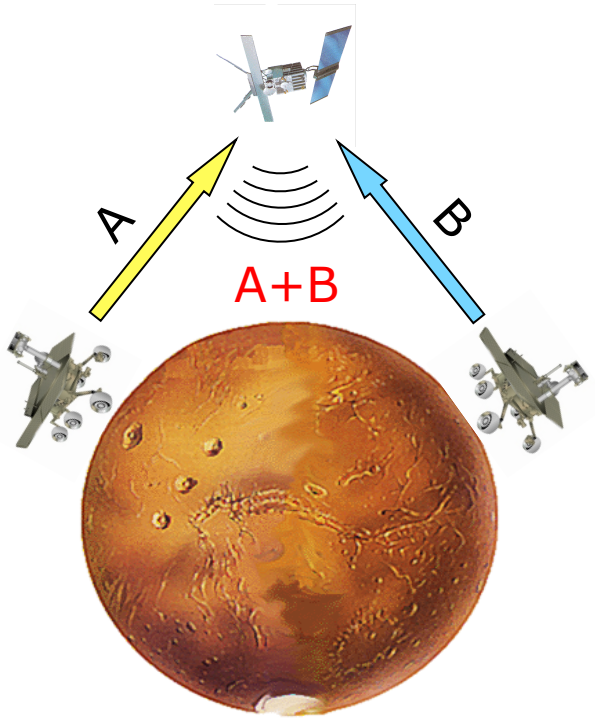

PNC (CUHK)

- Estimates A+B

ANC (MIT)

- Amplify and forward

Concluding Remarks

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- A new information infrastructure for transmission, storage, security, etc, is underway
- Network coding will continue to interact with different fields of research

Thank you